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SEARCH FOR EXPERIMENTAL EVIDENCE ON EXCLUSIVE DOUBLE-POMERON EXCHANGE *

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Abstract: Experience with single diffraction is used to motivate a proposed definition for the phase-space region of exclusive double-pomeron exchange (DPE); the definition involves two ratios of missing-mass to total energy. The kinematical implications of the proposed definition are explored through a triangle plot in Z variables – the logarithms of these ratios – and the problem of background is analyzed through a double-Regge expansion. It is shown that forthcoming NAL experiments should have no difficulty in establishing the presence or absence of exclusive double pomeron exchange. The results of previous attempts to measure DPE are reconsidered in terms of the Z variables, and it is found that the statistics accumulated to date are inadequate. Recent 205 GeV/c NAL experiments on $\pi^- p \to p\pi^-\pi^+\pi^-$ and $pp \to pp\pi^+\pi^-$ are discussed in some detail.

1. Introduction

In recent years many analyses of experimental data have sought evidence for multi-Regge behavior of high-energy reaction amplitudes and inclusive cross sections [1], the number-one objective being verification of double-pomeron exchange [2]. Controversy continues to surround the nature of the pomeron, its capacity to appear more than once in a single amplitude being doubted by those who regard the pomeron not as a Regge pole but merely as a synonym for "diffraction". In spite of the importance of the question, there has been remarkable lack of agreement among particle physicists as to what constitutes a suitable experimental test for the presence (or absence) of double-pomeron exchange. In this paper, by reviewing already established information on single-pomeron exchange, we are led to propose definitive criteria for testing the double-pomeron hypothesis.

Pomeron exchange is definable either in an exclusive or in an inclusive sense [3] – as one recognizes immediately in the original application to differential elastic as well as to total cross sections. Double-pomeron exchange may correspondingly refer to double exclusive, double inclusive or single-inclusive–single-exclusive. We

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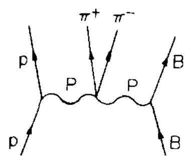

Fig. 1. Schematic representation of the double-pomeron contribution to the amplitude for reaction (1): $Bp \to p\pi^+\pi^-B$.

concentrate in this paper on double-exclusive measurements – for three reasons:

(i) Much more attention has been devoted to data relevant to the other two categories from which, despite ambiguities of interpretation, it is now widely accepted [1] that double-pomeron effects are indicated *.

(ii) Theoretical skepticism about multiple-pomeron effects in the exclusive sense seems sharper than for the inclusive.

(iii) Data relevant to double-pomeron exchange in the exclusive sense is more difficult to accumulate and more care is correspondingly needed in the analysis.

Most work to date on the double-exclusive question [4, 5] has employed reactions of the type

$$Bp \to B\pi^+\pi^- p\ , \quad (B = \pi \text{ or } p)\ , \tag{1}$$

where there may occur a double-pomeron exchange contribution to the amplitude as schematically indicated in fig. 1. We shall begin this paper by reviewing the literature on such reactions and stressing the absence of a uniformly accepted criterion for establishing the double-pomeron effect. We then consider a criterion that has become accepted in studying single-pomeron (exclusive) effects and examine the consequences of employing the corresponding criterion for double-pomeron exchange. Although our conclusion from such a criterion is that no experiment to date yields significant evidence for or against exclusive double-pomeron effects, we are able to spell out the requirements for meaningful experiments feasible with present accelerators. We discuss several models that are useful in data analysis and review previous work in these terms.

2. What is a "pomeron-associated event"?

Table 1 lists the published experiments on reactions of the type (1) and the type of analysis used to define double-pomeron "events" [4, 5]. In each case a certain portion of phase space was identified as being the region where the double-pomeron

* The double-inclusive question is usually phrased as the presence or absence of a central energy-independent plateau in an inclusive distribution. The single-inclusive–single-exclusive question is posed as the presence or absence of a PPP term in a triple-Regge expansion.

Table 1
Part I = (πp) experiments

Reaction	P_{lab} (GeV/c)	Ref.	Selection criteria for (DPE) candidates	Observation and claimed result
$\pi^+ p \to p\pi^- \pi_s^+ \pi_f^+$	8 and 16		• longitudinal phase-space analysis: the angle ω for the $(\pi^+\pi_s^-)$ system of (DPE) candidates is $\simeq 120°$.	Superposition of such (DPE) candidates with the tails of other phenomena.
$\pi^+ p \to p(2\pi^0)\pi^+$		[4a]	• $\left.\begin{matrix} M_{\pi^-\pi^+_{\text{slow}}} \\ M_{\pi^0\pi^0} \end{matrix}\right\} < 0.65$ GeV.	No conclusion can be reached regarding: (i) the existence of (DPE) (ii) the energy dependence of (DPE)
$\pi^- p \to p\pi^+ \pi_s^- \pi_f^-$	11 and 16	[4b]	• longitudinal phase-space analysis: the angle ω for the $(\pi^+\pi_s^-)$ system of (DPE) candidates is $\simeq 120°$.	same as above
$\pi^- p \to p\pi^+ \pi_s^- \pi_f^-$	25	[4c]	• $M^2_{(\pi X)} \geqslant 2$ GeV^2, $M^2_{(pX)} \geqslant 4$ GeV^2 • $m_X^2 \leqslant 2$ GeV^2	(DPE) is either severely suppressed or absent.
			• $\lvert t_{\pi\to\pi_f} + 2t_{p\to p}\rvert \leqslant 0.8$ GeV^2	$\sigma_{\text{upper limit}} \simeq 10\,\mu$b
$\pi^- p \to p\pi^+ \pi_s^- \pi_f^-$	205	[4d]	• $y_{AX} \simeq Z_A - 1 \geqslant 2, y_{BX} \simeq Z_B \geqslant 2$	(12 ± 3.5) events corresponding to $\sigma = (45 \pm 13)\,\mu$b (and $\sigma_{\text{upper limit}} = 65\,\mu$b)
			• a fit based on a multi Regge model [12] of the density inside the triangle y_{AX} versus y_{BX}	(16 ± 12) events when (DPE) term included
			• selection based on a pion pole dominance model [17] $M^2_{\pi^+\pi_{\text{fast}}} \geqslant 2$ GeV^2, $M^2_{p\pi} \geqslant 4$ GeV^2, $y_{\pi_s^-\pi_f^-} \geqslant 2$	8 events – subset of the 12 – corresponding to $30 \pm 10\,\mu$b, to be compared with 34 μb predicted by a pion pole dominence model [17].

Table 1
Part II = (pp) experiments

Reaction	P_{lab} (GeV/c)	Ref.	Selections criteria for (DPE) candidates	Observation and claimed result
$pp \to p\pi^+\pi^-p$	4–25 (pp World DST)	[5a]	• longitudinal phase-space analysis	• Small energy dependence in the central region found consistent with a sizable (DPE) effect, but limited to the high energy range of 19–25 GeV/c.
	4–25	[5b]	• uses double Regge model [18] to select events	• For the energy range considered, pomeron-reggeon exchange is adequate to explain the data without any contribution from double-pomeron exchange.
			• $M_{p\pi\pi} > 1.7$ GeV, $\lvert\cos\theta^*\rvert \geqslant 0.9$ (θ^*, angle between incoming and corresponding outgoing proton)	• A spin-parity analysis of the $\pi\pi$ system indicates a substantial P-wave contribution arguing against (DPE) dominance.
	19.22. 25 (from pp World DST)	[5c]	• assumption that the ($\pi\pi$) system is in a pure S-wave. • uses a double Regge model to make prediction on $M(\pi\pi)$ inside LPS region for (DPE).	• No evidence for any large (DPE) contributions.
	12 and 24	[5d]	• $\lvert y^*_{\pi^+\pi^-}\rvert < 0.5$	• Observation of an enhancement in the low (2π) mass region.
			• $M_X < 0.6$ GeV	• This low mass is completely dominated by fragmentations and/or excitation of the incident protons.
				• $\sigma_{\text{upper limit}}$ at 24 GeV/c = 30 μb.
	205	[5e]	• selection based on a pion pole dominence model [17] $M^2_{p\pi} \geqslant 4$ GeV^2	• 9 events $\to \sigma = 44 \pm 15$ μb in agreement with the prediction of 31 μb from a pion pole dominence model [17]

Table 1 (*continued*)

Reaction	P_{lab} (GeV/c)	Ref.	Selections criteria for (DPE) candidates	Observation and claimed result
		[5e]	• $M_X < 0.6$ GeV also required	• 2 events → $\sigma = 9\,\mu$b conclusion: $\sigma_{\text{upper limit}} =$ $= 44\,\mu$b and no evidence of (DPE).

mechanism had the best chance to show itself. But the choice of this region varied from one experiment to another as did the efforts to estimate "background".

The principles of quantum mechanics preclude any precise basis for associating a given event with pomeron exchange, but experience with single (exclusive) pomeron exchange has led to widespread use of the concept of "diffractive" events. Although this concept cannot be precise, it is useful and has become understood by particle physicists in a fairly uniform sense; the concept is equivalent to a definition of a "pomeron-associated event". The most natural definition of a "double-pomeron event" in the reaction (1) is then, to interpret fig. 1 as either single diffraction of the type shown in fig. 2a or as single diffraction of the type shown in fig. 2b and to demand that an event simultaneously satisfy the conventional criteria for both. In order to implement this definition of DPE, we must now identify in quantitative terms the common understanding of what constitutes single diffraction.

Extensive high-energy inclusive experiments of the type Bp → pX (B = p, π, K) have led to the characterization of events, for which the absolute value of the Feynman variable

$$|x_p| \approx 1 - M_{Xp}^2/s \tag{2}$$

is larger than about 0.9, as "diffractive" [1, 6]. The symbol M_{Xp} stands for the missing mass with respect to the proton while s is the square of the total c.m. energy. Within the restricted interval $M_{Xp}^2/s \lesssim 0.1$ two qualitative characteristics associated with "diffraction" have been observed [6] : (i) The dependence of $d\sigma/dM_{Xp}^2$ on momentum transfer to the proton is steep – similar to that in elastic scattering. (ii) The dependence on s of $d\sigma/dM_{Xp}^2$ is weak – again similar to elastic scattering.

It is illuminating to recognize the connection between the ratio M_{Xp}^2/s and the rapidity gap y_{pX} between the final observed proton and its nearest neighbor among the remaining produced particles of mass M_{Xp}. It has been shown [7] that for large s such a gap y_{pX} is related to M_{Xp}^2/s on a statistical basis by *

* Assuming the particle within X that is closest to p to be a π.

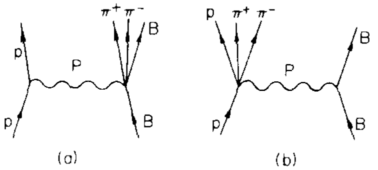

Fig. 2. Single-diffractive interpretation of fig. 1.

$$y_{\mathrm{pX}} \approx \ln \frac{s}{M^2_{\mathrm{Xp}}} + \ln \frac{\langle m_{\perp\pi}\rangle}{\langle m_{\perp\mathrm{p}}\rangle} , \tag{3}$$

(where $\ln\,[\langle m_{\perp\pi}\rangle/\langle m_{\perp\mathrm{p}}\rangle] \approx -1$ if the average transverse momentum of produced particles is ≈ 350 MeV/c). The requirement that M^2_{Xp}/s be small thus means that y_{pX} be large – the qualitative condition for pomeron dominance given by Regge theory [8].

We thus propose a preliminary definition of a "double-pomeron event" of the type AB → AXB (see fig. 1) as one for which $1 - |x_{\mathrm{A}}| \lesssim 0.1$ and $1 - |x_{\mathrm{B}}| \lesssim 0.1$, where

$$|x_{\mathrm{A}}| \approx 1 - \frac{M^2_{\mathrm{XA}}}{s} , \tag{4}$$

$$|x_{\mathrm{B}}| \approx 1 - \frac{M^2_{\mathrm{XB}}}{s} . \tag{5}$$

By such a definition, DPE events constitute a part of single diffraction dissociation, but each event may be described as dissociation either of A or B and belongs simultaneously to both singly diffractive regions.

Although the definition of DPE is given in terms of x_{A} and x_{B}, an important kinematic constraint is more easily recognized if one thinks in terms of the corresponding rapidity gaps y_{AX} and y_{BX}. The sum $y_{\mathrm{AX}} + y_{\mathrm{BX}}$ evidently cannot be greater than the gap y_{AB} between the outgoing particles A and B, while y_{AB} is limited * by s:

$$\langle y_{\mathrm{AB}}\rangle \approx \ln \frac{s}{\langle M_{\perp\mathrm{A}}\rangle \langle m_{\perp\mathrm{B}}\rangle} . \tag{6}$$

* Relation (6) has been verified for (πp) experiments, but not for pp experiments where its application would give

$$y^{\mathrm{incoming}}_{\mathrm{AB}} - \langle y_{\mathrm{AB}}\rangle \simeq \ln \frac{\langle m_{\perp\mathrm{p}}\rangle}{m_{\mathrm{p}}} \simeq 0.13 .$$

Experimentally, one finds this difference to be of the order of ≈ 1 unit (see ref. [5c, 5e]). We will nevertheless, for reasons of simplicity, continue to use formula (6).

We thus have:

$$y_{AX} + y_{BX} < \ln \frac{s}{\langle m_{\perp A}\rangle \langle m_{\perp B}\rangle},$$

or using relations (3)–(5) where the index p is replaced by A and B respectively:

$$\ln \frac{1}{(1-x_A)(1-x_B)} < \ln \frac{s}{s_0}. \tag{7}$$

If the methods of ref. [7] which led to formula (3) are applied, one finds that s_0 is independent of the particles A and B and is of the order of magnitude $\langle m_\perp \rangle^2$, where $\langle m_\perp \rangle$ is the mean transverse mass, $[m^2 + \langle p_\perp^2 \rangle]^{\frac{1}{2}}$, of the nearest neighbor to A or B. Assuming such a particle to be a pion one expects

$$s_0 \approx 0.14 \text{ GeV}^2 . \tag{8}$$

3. The triangle plot for double-pomeron events

The foregoing arguments suggest the introduction of variables

$$Z_A \equiv \ln \frac{s}{M_{XA}^2} \approx \ln \frac{1}{1-x_A},$$

$$Z_B \equiv \ln \frac{s}{M_{XB}^2} \approx \ln \frac{1}{1-x_B}. \tag{9}$$

which are equivalent to rapidity gaps, up to displacements of the order of 1 [7]. These two variables span a triangular region of phase space as shown in figs. 3(a) and 3(b), being limited by the constraint

$$Z_A + Z_B < \ln (s/s_0), \qquad s_0 \approx 0.14 \text{ GeV}^2 . \tag{10}$$

We are defining double-pomeron events as those which fall into the region where

$$e^{-Z_A} \lesssim 0.1 , \qquad e^{-Z_B} \lesssim 0.1 , \tag{11}$$

or

$$Z_A \gtrsim 2.3 , \qquad Z_B \gtrsim 2.3 . \tag{12}$$

One sees by this definition how the region of possible DPE events expands with increasing total energy.

A useful feature of the triangle plot, in addition to its geometrical simplicity, is

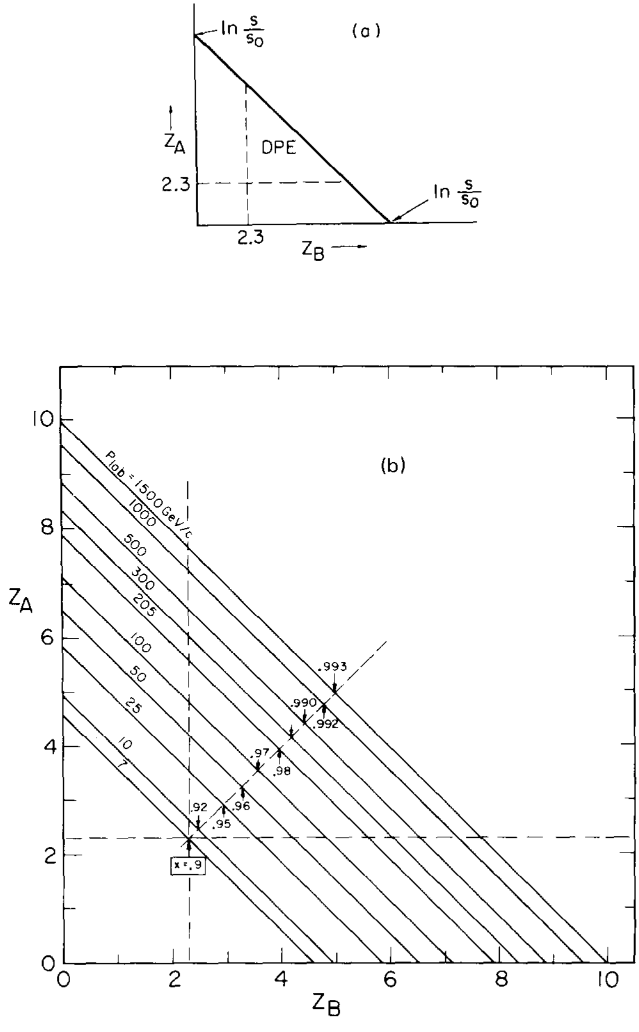

Fig. 3(a).The triangle plot defining the double-pomeron region. (b) The triangle plot for different values of P_{lab}. Note the maximum value of $|x_A|$ and $|x_B|$ when these variables are constrained to be equal to each other.

that, at high energy, equal areas within the triangle correspond to equal regions of "multiperipheral phase space." This statement will made precise in sect. 5 when we consider the question of multi-Regge analysis. For the moment we merely remark that the linear expansion with $\ln s$ of the DPE region in fig. 3 implies a parallel increase in the expected number of DPE events.

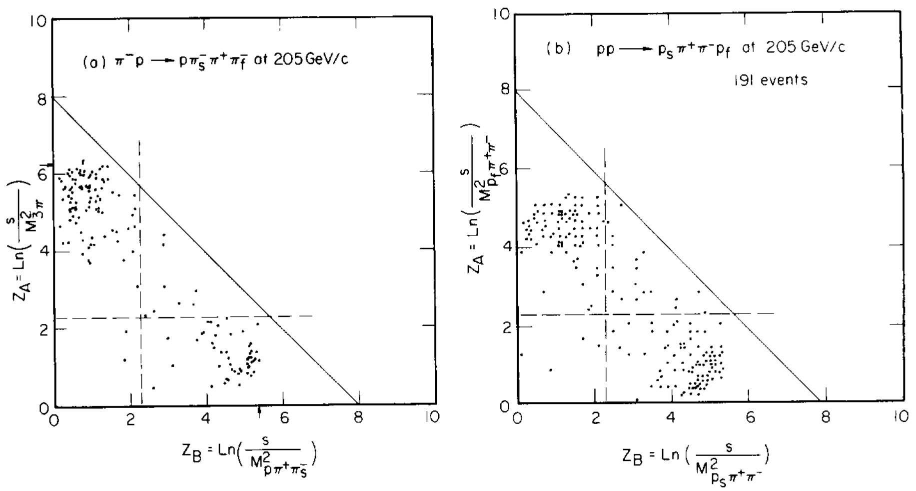

Fig. 4. (a) The triangle plot of fig. 3 with events of the reaction $\pi^- p \to p\pi^+\pi_s^-\pi_f^-$ at 205 GeV/c. (b) The triangle plot of fig. 3 with events of the reaction $pp \to p_s\pi^+\pi^-p_f$ at 205 GeV/c.

The larger s is, the more favorable are the conditions for observing DPE. Fig. 3 shows that the absolute minimum s for DPE observation is given by

$$\ln \frac{s}{s_0} \approx 2(2.3)$$

or

$$s \approx 100\, s_0$$
$$\approx 14 \text{ GeV}^2 .$$

The largest value of s for which reactions of type (1) have been studied to date is ≈ 400 GeV^2 at NAL, corresponding to $\ln(s/s_0) \approx 8$, so the DPE region here is substantial. At the ISR one can reach $\ln(s/s_0) \approx 10$ (fig. 3b).

4. Currently available data

NAL experiments with 205 GeV/c pions and protons have each generated only a handful of events in the DPE region [4d, 5]. The triangle plot of events from the reaction $\pi^- p \to \pi^-\pi^+\pi^- p$ is shown [7] in fig. 4a. The great majority of the events lie in regions where either Z_A or Z_B is large, but not both. These are the singly diffractive events. The eight events that are DPE by our definition correspond to a cross section of 30 ± 11 μb. Results from the reaction $pp \to pp\pi^+\pi^-$ are similar [10a] (fig. 4b). The selection of 17 events of the pp experiment would correspond, using the information of ref. [10b], to 60 ± 15 μb. The factorizability of the pomeron (see eq. (14)) leads one to expect that the ratio of DPE cross sections in pp and πp collisions is approximately equal to the ratio of the corresponding elastic cross sections (≈ 2).

Experiments at lower energies have no better statistics in the DPE region so it will suffice to ask whether the presently available 205 GeV/c results do or do not establish the existence of double-pomeron exchange. In other words, can the 8 (or 19) events be no more than "background" from the tails of distributions concentrated in the single diffraction regions of the triangle plot? A visual estimate suggests that such could easily be the case; in ref. [4d] a simple Regge fit to the overall distribution confirmed the statistical insignificance of the selected events in the πp experiment.

5. A formula for double-Regge analysis

Supposing that meaningful statistics were available, how would one proceed to establish the presence or absence of double-pomeron exchange? Let us first analyze the problem in terms of the rapidity gaps y_{AX} and y_{BX} and later change to the equivalent Z variables. We assume that the two momentum-transfer variables t_{A} and t_{B} have also been measured.

At a fixed value of the total energy, if we sum over the variables of the internal cluster, the cross section is a function of four independent variables, $t_{\mathrm{A}}, t_{\mathrm{B}}, y_{\mathrm{AX}}$, and y_{BX}. The mass of the internal cluster is fixed by the difference between $y_{\mathrm{AX}} + y_{\mathrm{BX}}$ and the total rapidity interval y_{AB} as given by formula (6) in terms of s. Let us designate by y_{X} the rapidity interval spanned by the central cluster *, so that

$$y_{\mathrm{AX}} + y_{\mathrm{BX}} + y_{\mathrm{X}} = y_{\mathrm{AB}} \,. \tag{13}$$

For large values of y_{AX} and y_{BX}, according to double-Regge theory, the differential cross section has an asymptotic expansion [12]

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}t_{\mathrm{A}}\mathrm{d}t_{\mathrm{B}}\mathrm{d}y_{\mathrm{AX}}\mathrm{d}y_{\mathrm{BX}}} \approx \sum_{i,j,k,l} \beta_{i\mathrm{A}}(t_{\mathrm{A}})\,\beta^*_{j\mathrm{A}}(t_{\mathrm{A}})\,\mathrm{e}^{[\alpha_i(t_{\mathrm{A}})+\alpha_j(t_{\mathrm{A}})-2]y_{\mathrm{AX}}}$$

$$\times\, g_{ij,kl}(y_{\mathrm{X}}, t_{\mathrm{A}}, t_{\mathrm{B}})\,\mathrm{e}^{[\alpha_k(t_{\mathrm{B}})+\alpha_l(t_{\mathrm{B}})-2]y_{\mathrm{BX}}}\,\beta_{k\mathrm{B}}(t_{\mathrm{B}})\,\beta^*_{l\mathrm{B}}(t_{\mathrm{B}}) \,, \tag{14}$$

corresponding to fig. 5, the sum running over all Regge trajectories with zero quantum numbers. Our immediate goal is to establish whether any four-reggeon coupling $g_{ij,kl}$ is non-vanishing for which at least one of the two indices ij corresponds to a pomeron and simultaneously at least one of the two indices kl is also a pomeron.

* The mass squared of the central cluster is roughly equal to $s_0\,\mathrm{e}^{y_{\mathrm{X}}}$ as shown in formula (29).

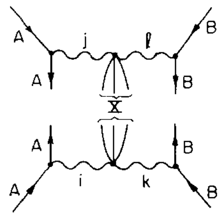

Fig. 5. Schematic representation fo formula (14).

Ultimately, of course, the individual values of the various four-reggeon couplings will become a goal.

With sufficient statistics the analysis can proceed for fixed values of t_A and t_B, or one may integrate over these variables and replace each α by an appropriate t average. In either case let us now drop further reference to t_A and t_B and concentrate on the Regge dependence on y_{AX} and y_{BX} exhibited by formula (14).

Exploitation of this simple Regge dependence, which is to be the basis of our analysis, requires that y_X be kept fixed. Keeping the constraint (13) in mind, it is convenient to define

$$y \equiv \tfrac{1}{2}(y_{AX} - y_{BX}) \,, \tag{15}$$

so that

$$y_{AX} = \tfrac{1}{2}(y_{AB} - y_X) + y \,, \quad y_{BX} = \tfrac{1}{2}(y_{AB} - y_X) - y \,. \tag{16}$$

We may then rewrite formula (14) as

$$\frac{d^2\sigma}{dy_x dy} \approx \sum_{ij,kl} G^{AB}_{ij,kl}(y_X)\, e^{\frac{1}{2}[\alpha_i + \alpha_j + \alpha_k + \alpha_l - 4]y_{AB} + [\alpha_i + \alpha_j - \alpha_k - \alpha_l]y} \,. \tag{17}$$

At this stage a change is easily made to the variables Z_A and Z_B, defining by analogy to (15)

$$Z \equiv \tfrac{1}{2}(Z_A - Z_B)$$

$$= \ln(M_{XB}/M_{XA}) \,. \tag{18}$$

Remembering the relation (6) as well as the fact that the Z and y variables are related by a simple displacement, we rewrite (17) as

$$\frac{d^2\sigma}{dZ_X dZ} \underset{\substack{Z_A \text{ and } Z_B \\ \text{both "large"}}}{\approx} \sum_{ij,kl} G^{AB}_{ij,kl}(Z_X)\, s^{\frac{1}{2}[\alpha_i + \alpha_j + \alpha_k + \alpha_l - 4]}\, e^{[\alpha_i + \alpha_j - \alpha_k - \alpha_l]Z} \,, \tag{19}$$

where *

$$Z_X \equiv \ln \frac{s}{s_0} - (Z_A + Z_B) . \tag{20}$$

Formula (19) is now suitable for use in conjugation with the triangle plot.

Implementation of formula (19) is made easier by using a slightly different plot than that of fig. 3, now choosing Z as the horizontal axis and Z_X as the vertical axis. Data at a particular energy then fall within an isosceles triangle whose base and alitude are both equal to $\ln (s/s_0)$ as illustrated in fig. 6a, b **. The values of Z_A and Z_B for an event point within the triangle are proportional to the perpendicular distances to the two sides of the triangle, so the validity of formula (19), which requires both Z_A and Z_B to be large, is restricted to the central lower region. The dotted lines in fig. 6, for example, delineate the domain where both Z_A and Z_B are larger than 2.3, that is, the region labeled DPE in fig. 3.

Formula (19) shows that if for some range of Z and Z_X within the central lower region the cross section is found to be independent of s, one will have established exclusive double-pomeron exchange. That is, since no α can be larger than 1, absence of s-dependence can only be achieved by the dominance of a term where $\alpha_i \approx \alpha_j \approx \alpha_k \approx \alpha_l \approx 1$. At the same time, according to formula (19), such complete pomeron dominance implies an absence of dependence on Z. By itself, of course, the latter observation would not be proof of double-pomeron exchange.

In practice one expects a substantial role for secondary Regge poles, so let us now look at the "background" that tends to obscure double-pomeron exchange.

6. Simple models of background

So-called "triple-Regge" analysis often employs the fiction of a single secondary pole, labeled R, in addition to the pomeron, labeled P. If we [13] do the same and take $\alpha_P = 1$, formula (19) becomes

$$\frac{d^2\sigma_{AB}}{dZ_X dZ} \approx G^{AB}_{PP,\,PP}(Z_X) + G^{AB}_{PP,\,\overline{PR}}(Z_X)\, s^{-\frac{1}{2}(1-\alpha_R)}\, e^{(1-\alpha_R)Z}$$

$$+ G^{AB}_{\overline{PR},\,PP}(Z_X)\, s^{-\frac{1}{2}(1-\alpha_R)}\, e^{-(1-\alpha_R)Z} + G^{AB}_{\overline{PR},\,\overline{PR}}(Z_X)\, s^{-(1-\alpha_R)}$$

$$+ G^{AB}_{PP,\,RR}(Z_X)\, s^{-(1-\alpha_R)}\, e^{+2(1-\alpha_R)Z} + G^{AB}_{RR,\,PP}(Z_X)\, s^{-(1-\alpha_R)}\, e^{-2(1-\alpha_R)Z}$$

* It is shown below (formula (36)) that $Z_X \approx \ln (M_X^2/s_0)$ where M_X is the mass of the central cluster.

** In ref. [4d] the triangle was made equilateral by taking $|y_{AX} - y_{BX}|/\frac{1}{2}\sqrt{3}$ for the horizontal axis.

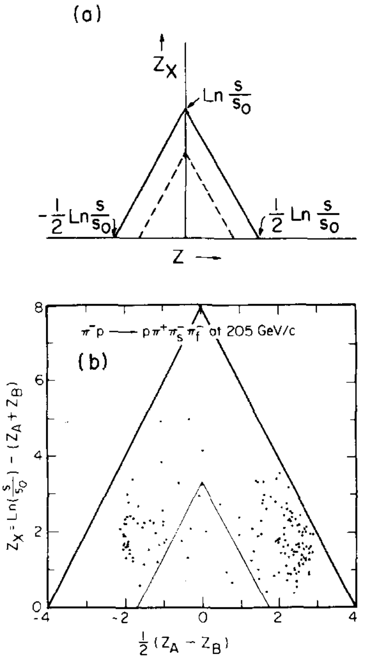

Fig. 6. (a) Isosceles triangle plot. (b) Isosceles triangle plot with events of the reaction $\pi^- p \to p\pi^+\pi_s^-\pi_f^-$ at 205 GeV/c.

$$+ G^{AB}_{\overline{PR}, RR}(Z_X)\, s^{-\frac{3}{2}(1-\alpha_R)}\, e^{(1-\alpha_R)Z} + G^{AB}_{RR,\overline{PR}}(Z_X)\, s^{-\frac{3}{2}(1-\alpha_R)}\, e^{-(1-\alpha_R)Z}$$

$$+ G^{AB}_{RR, RR}(Z_X)\, s^{-2(1-\alpha_R)}\,, \tag{21}$$

where the bar notation means, for example,

$$G^{AB}_{PP, \overline{PR}} = G^{AB}_{PP, PR} + G^{AB}_{PP, RP}\,. \tag{22}$$

Even though the last three terms in (21) may be negligible at NAL energies, it will almost certainly be impossible to determine all six remaining coefficients. Formula (21) nevertheless exhibits a simple criterion for the presence of some double-pomeron contributions: an s dependence that falls more slowly than $s^{-(1-\alpha_R)}$. Considering the fact that α_R represents a t average, we expect $\alpha_R = \approx 0.3$ so our criterion is an s-dependence of the cross section for events within fixed intervals of Z and Z_X that falls more slowly than $\propto s^{-0.7}$.

What effective s-power law might one expect to find in the NAL range if double-pomeron effects do not vanish? An alternative to the P, R model, suggested by Dash [14] for triple-Regge application, uses a single Regge pole that represents the average effect of P and R. Designating such a pole by P_0 and its position by α_0, one has from formula (19)

$$\frac{d^2\sigma_{AB}}{dZ_X dZ} \approx G^{AB}_{P_0P_0,\, P_0P_0}(Z_X)\, s^{-2(1-\alpha_0)}\ . \tag{23}$$

Dash had success in fitting triple-Regge data with $\alpha_0 \approx 0.85$, so for the cross section considered here one anticipates an effective power behavior $\propto s^{-0.3}$. Experiments at NAL should have no difficulty in distinguishing $s^{-0.3}$ from $s^{-0.7}$. If the result is closer to the former than to the latter, double pomeron exchange will have been established in the exclusive sense. At the ISR, with an additional factor of 10 in s, the leading term in formula (21) may stand out sufficiently that a value can be determined for $G^{X}_{PP,\,PP}$.

According to formula (21), useful information resides in the Z dependence as well as the s-dependence, although the former is less decisive in establishing double-pomeron behavior. A popular triple-Regge model ignores interference terms (terms carrying barred indices) and it is interesting to make such a simplification in formula (21), at the same time dropping the term where no pomeron appears:

$$\frac{d^2\sigma_{AB}}{dZ_X dZ} \approx G^{AB}_{PP}(Z_X) + G^{AB}_{PR}(Z_X)\, s^{-(1-\alpha_R)}\, e^{2(1-\alpha_R)Z}$$

$$+\, G^{AB}_{RP}(Z_X)\, s^{-(1-\alpha_R)}\, e^{-2(1-\alpha_R)Z}\ . \tag{24}$$

The two "background" terms may be identified with the two single-diffractive mechanisms indicated in fig. 2, one term tending to populate the region near the left-hand side of the triangle and the other populating the region near the right-hand side.

The formula (24) was used to fit the Z-dependence of the 205 GeV $\pi^- p \to \pi^-\pi^+\pi^- p$ data discussed above [4d], and it was found that the best value of α_R was close to 0.5, rather than the anticipated 0.3. This fact probably reflects the importance of the neglected interference terms. In any event, the magnitude of the background indicated by this fit was such as to allow only an upper limit determination of the coefficient $G^{\pi p}_{PP}$. The integral of the corresponding term over the entire triangle corresponds to 9 ± 8 events [4d], a number which – though not statistically significant – is comparable with the 8 events inside the inner triangle of fig. 4a.

7. Comparison with previous definitions of (DPE)

7.1. Kinematics

Previous definitions of DPE have used a variety of cuts on masses and (or) momentum transfer, as well as rapidity cuts. Let us see how the Z variables proposed here are related to previously studied variables.

First we note that the requirement $Z_A(Z_B) \geqslant 2.3$ is equivalent to demanding that $M_{XA}(M_{XB})$ be less than $\sqrt{s/10}$.

More drastic definitions of single diffraction (placing a lower limit on $|x|$ bigger than 0.9) would give the cuts on M_{XA} (and M_{XB}) shown in fig. 7. In this figure, the darker line represents (vs P_{lab}) the maximum value reachable by $|x_A|$ and $|x_B|$ when these variables are constrained to be equal to each other (fig. 3). Figs. 8a, 8b show the masses M_{XA} and M_{XB} for π^-p and pp at 205 GeV/c, and the selection of (DPE) candidates corresponding to Z_A and $Z_B \geqslant 2.3$.

A rough statistical correspondence exists between $Z_A(Z_B)$ and the combined mass $M_{A\pi}(M_{B\pi})$ of particle A (B) together with its nearest neighbor within the missing mass M_X. Starting with the general formula for a two-particle combination

$$S_{ij} = M_{ij}^2 = m_i^2 + m_j^2 + 2m_{i\perp} m_{j\perp} \cosh(y_i - y_j) - p_{i\perp} \cdot p_{j\perp} , \tag{25}$$

and assuming $|y_\pi - y_A|$ sufficiently large that

$$2 \cosh(y_\pi - y_A) \approx \exp|y_\pi - y_A| ,$$

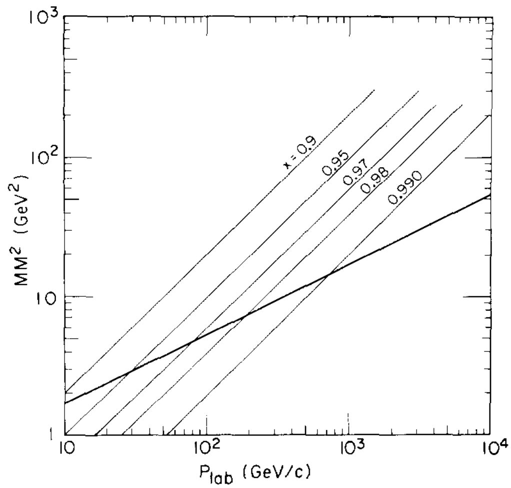

Fig. 7. M_{XA}^2 (M_{XB}^2) *versus* P_{lab} for different values of x; the darker line across the lines of x corresponds to the maximum value of $|x_A|$ and $|x_B|$ as obtained from fig. 3.

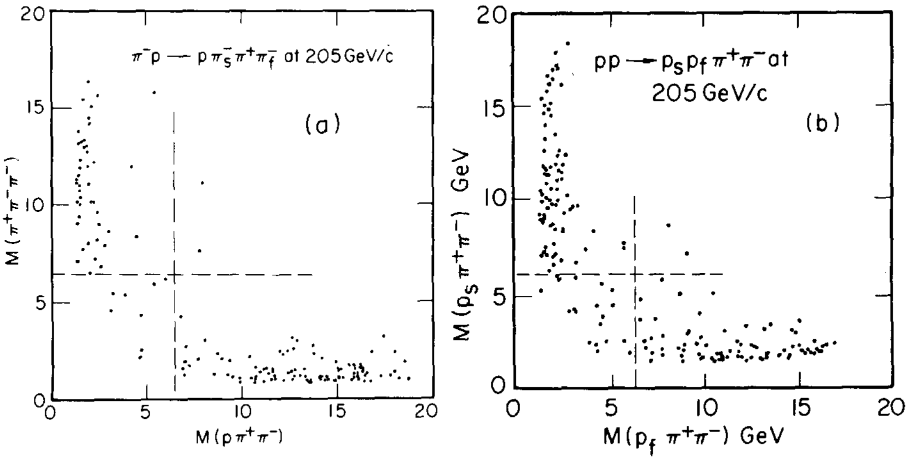

Fig. 8. (a) M_{XA} *versus* M_{XB} with events of the reaction $\pi^- p \to p\pi^+\pi_s^-\pi_f^-$ at 205 GeV/c. (b) M_{XA} *versus* M_{XB} with events of the reaction $pp \to p_s\pi^+\pi^- p_f$ at 205 GeV/c.

and also that $p_{\perp\pi} \cdot p_{\perp A}$ averages to zero, we have

$$|y_\pi - y_A| \approx \ln \frac{M_{A\pi}^2 - m_A^2 - m_\pi^2}{m_{\perp A} m_{\perp\pi}} . \tag{26}$$

In ref. [7] it was shown that on a statistical basis

$$|y_\pi - y_A| \approx Z_A + \ln \frac{\langle m_{\perp\pi}\rangle}{\langle m_{\perp A}\rangle} . \tag{27}$$

Combining (26) and (27) we thus obtain

$$Z_A \approx \ln \frac{M_{A\pi}^2 - m_A^2 - m_\pi^2}{\langle m_{\perp\pi}\rangle^2} , \tag{28}$$

with a corresponding formula for Z_B. DPE events must be such that $M_{p\pi} \gtrsim 1.5$ GeV and $M_{\pi\pi\,\mathrm{fast}} \lesssim 1.20$ GeV.

Fig. 9a exhibits these s-independent relations and figs. 10 a, b use events from the reaction $\pi^- p \to p\pi_s^-\pi^+\pi_f^-$ at 205 GeV/c to demonstrate that, despite wide event fluctuations, (28) works fairly well in an average sense.

The mass M_X of the two-pion central cluster is roughly related to the sum $Z_A + Z_B$. To find this relation we start with the general formula (25) applied to the two-pion combination and find, corresponding to (26),

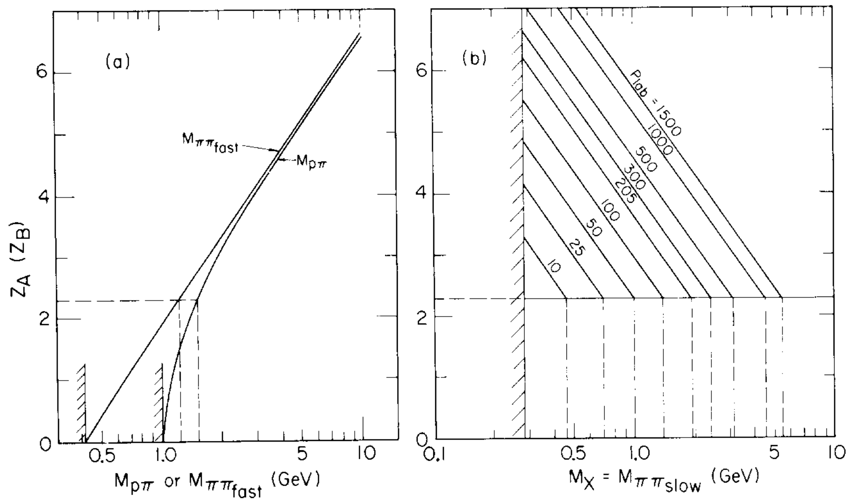

Fig. 9. (a) Z_A (or Z_B) *versus* $\langle M_{A\pi}\rangle$ (or $\langle M_{B\pi}\rangle$) for A (or B) = π or p, according to the s-independent formula (28). (b) Z_A (or Z_B) *versus* $\langle M_X\rangle$ according to formula (34) for different values of P_{lab}.

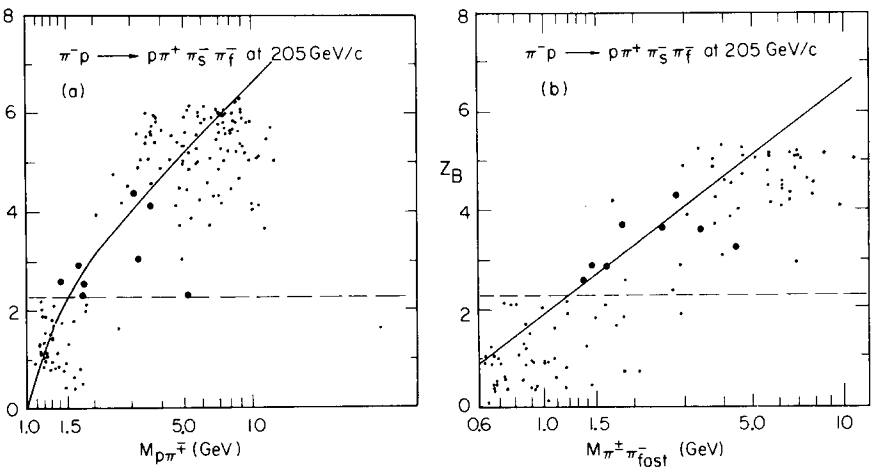

Fig. 10. (a) Z_A *versus* $M_{p\pi}$ for events of reaction $\pi^- p \to p\pi^+\pi_s^-\pi_f^-$ at 205 GeV/c. (b) Z_B *versus* $M_{\pi\pi_{fast}^-}$ for events of reaction $\pi^- p \to p\pi^+\pi_s^-\pi_f^-$ at 205 GeV/c. The full lines are illustrations of the s-independent formula (28). The large dots correspond to (DPE) events.

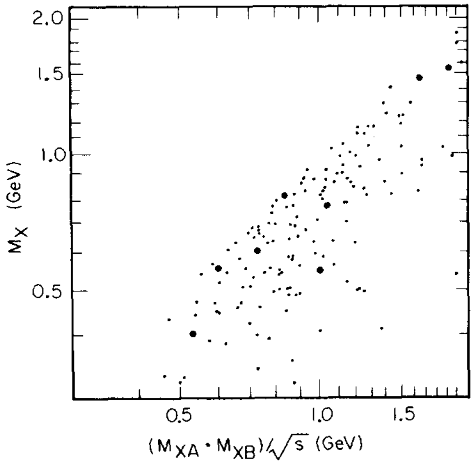

Fig. 11. M_X *versus* $M_{XA}M_{XB}/\sqrt{s}$ for events of the reaction $\pi^- p \to p\pi^+\pi_s^-\pi_f^-$ at 205 GeV/*c*. The large dots correspond to (DPE) events.

$$y_X \approx \ln \frac{M_X^2}{\langle m_{\perp\pi}\rangle^2} , \tag{29}$$

if y_X is the rapidity gap between the two pions. At the same time

$$y_X = y_{AB} - y_{AX} - y_{BX} , \tag{30}$$

while

$$y_{AB} \approx \ln \frac{s}{\langle m_{\perp A}\rangle \langle m_{\perp B}\rangle} , \tag{31}$$

$$y_{AX} \approx Z_A + \ln \frac{\langle m_{\perp\pi}\rangle}{\langle m_{\perp A}\rangle} , \tag{32}$$

$$y_{BX} \approx Z_B + \ln \frac{\langle m_{\perp\pi}\rangle}{\langle m_{\perp B}\rangle} . \tag{33}$$

It follows that

$$\ln \frac{s}{M_X^2} \approx Z_A + Z_B , \tag{34}$$

or equivalently ,

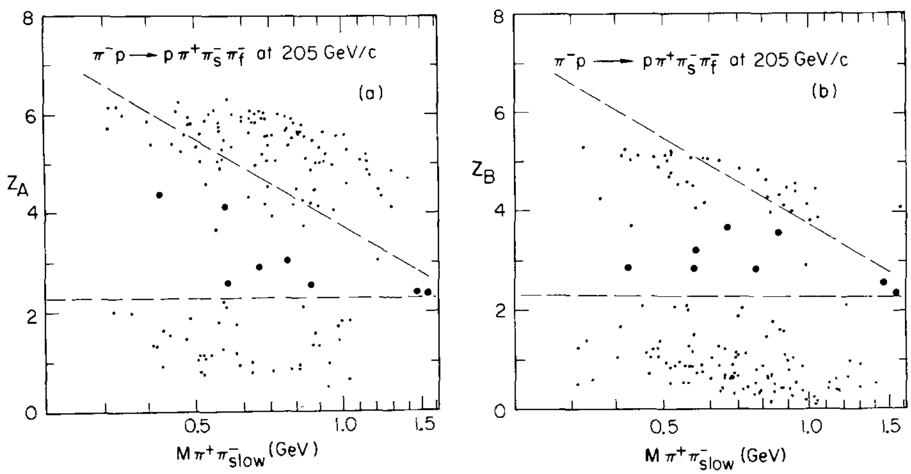

Fig. 12. (a) Z_A *versus* $M_{\pi^+\pi^-\text{slow}}$ for events of the reaction $\pi^-p \to p\pi^+\pi_s^-\pi_f^-$ at 205 GeV/c. (b) Z_B *versus* $M_{\pi^+\pi^-\text{slow}}$ for events of the reaction $\pi^-p \to p\pi^+\pi_s^-\pi_f^-$ at 205 GeV/c. The full lines are illustrations of formulae (34). The large dots correspond to (DPE) events.

$$M_X^2 \approx \frac{M_{AX}^2 M_{BX}^2}{s} . \tag{35}$$

Fig. (11) shows a plot of M_X versus $M_{AX} M_{BX}/\sqrt{s}$ for the π^-p experiment at 205 GeV/c. We see that on the average these two quantities tend to be roughly equal.

To satisfy our definition of DPE, M_{AX}^2 and M_{BX}^2 must each be smaller than $s/10$. It follows from (35) that M_X^2 must not be larger than $s/100$, but a simple cut on M_X^2/s does not define DPE. A second ratio must also be specified. By combining formulas (34) and (20), one may deduce that

$$Z_X = \ln \frac{M_X^2}{s_0} , \tag{36}$$

showing that in the isosceles triangle plot (fig. 6), M_X^2 is determined by the vertical coordinate. The upper limit on M_X^2 within the DPE region corresponds to the upper vertex of the inner triangle.

Fig. 9b gives *versus* Z_A(or Z_B) the range of M_X allowed within the DPE region for different values of P_{lab}. One observes that M_X has to be rather low (< 1 GeV) for all possible experiments up to NAL energies. Note also that a mass cut on M_X does not select only (DPE) candidates inside the kinematically allowed region, but also many events where Z_B or Z_A is small. The condition that M_X be small is necessary but not sufficient.

Figs. 12, b show $Z_A(Z_B)$ and M_X for the π^-p experiment at 205 GeV/c*. The

* The events in the plots presented in ref. [15] have been further selected in the same way as in ref. [11]. (χ^2 > 15) removed.

eight selected events of fig. 4a are circled and effectively almost all are inside the expected average kinematical boundaries.

7.2. *Physics: momentum transfer distributions*

Our proposed criterion for DPE has been expressed in terms of the Z variables, independently of the form of the dependence on t_A and t_B. Pomeron factorization predicts a peaking at small $|t_A|$ and $|t_B|$ related to that in elastic scattering, but practically all high energy reactions exhibit such peaks, so they cannot easily be used as part of a systematic experimental definition of DPE. Earlier work [4c] has sometime attempted to employ t-dependence as part of a DPE criterion, but we shall ignore such considerations.

7.3. *The different analyses which have been performed*

Table 1 gives a summary of the reactions and momenta (columns 1, 2) of the study, the different cuts adopted (column 3) and the results (column 4) of each of these experiments. Table 2 translates into variables, Z_A, Z_B the different data of table 1 (column 3) and gives in column 4 the different kinematical limits of the experiment. Column 5 gives the information on DPE in terms of our criteria.

Before going into details, we observe that previous studies have based the definitions of (DPE) on (i) either the remark of Van Hove [16] regarding the region of longitudinal phase-space where (DPE) events should be observed, or (ii) the definition of single diffraction using the rapidity variables y_{AX} and y_{BX} [4d] or (iii) theoretical models [4c, 17].

We will now examine each of these approaches and relate them to the criteria here:

(i) Longitudinal phase-space [4a, b–5a, b, c, d]. Van Hove made the remark [16] that for (DPE) candidates both π's within the X combination should be almost at rest in the general center of mass (for such events one could choose for instance $-0.125 \lesssim x_{\pi \text{ within X}} \lesssim 0.125$), while at the same time, in accord with fig. 1, the slowest particle should be A and the fastest B.

However, the interpretation of the resulting low M_X as a guarantee that $(\pi^+\pi^-)$ is preferentially in an S-state has proved to be wrong: a study of angular momenta [5b] has shown that for the reaction $pp \to p_s p_f \pi^+\pi^-$ between 4 and 25 GeV/c, no more than 50% of the $(\pi^+\pi^-)$ pairs were in an S wave despite all the cuts applied to the events [5a, b], even for very low masses of the $(\pi\pi)$ system. The necessary but not sufficient DPE requirement of an S (or D) wave (which would exclude isospin $I = 1$) cannot be achieved by only the mass cut on M_X. This fact reinforces the conclusions reached in subsect. 7.1.

(ii) The rapidity variables y_{AX} and y_{BX} [4d]. In a study applied to the 205 GeV/c π^-p experiment, events were called (DPE) which had both y_{AX} and $y_{BX} \geqslant 2$.

A consequence of our presently proposed definition of single diffraction

Table 2
Comparison with our analysis of all data available on (DPE)

Reaction	P_{lab} (GeV/c)	Ref.	Cuts used for (DPE) selection: From the authors	Cuts used for (DPE) selection: Equivalent in our Set of variables	Kinematic boundary $\ln(s/s_0)$	Z_A and $Z_B \geqslant 2.3$: Maximum value of $M_{XA}(M_{XB})$*	Z_A and $Z_B \geqslant 2.3$: M_X^{**}	Comments
$pp \to p_s\pi^+\pi^-p_f$	4	5b		$Z_A \leqslant$, $Z_B \leqslant$ 0.96($\lvert x_A\rvert < 0.62$)	4.0	0.9	0.3	• Few events (or none) of the DPE region are selected. Most events are single diffraction only
	12	5d	$M_X < 0.6$ GeV		5.1	1.55	0.5	
	19	5a, b, c	Ref. 5b takes a cut	2.6($\lvert x_A\rvert < 0.926$)	5.6	1.95	0.6	
	22	5a, b, c	on $M_{p\pi\pi} > 1.7$ GeV $\longrightarrow$	2.7($\lvert x_A\rvert < 0.93$)	5.7	2.1	0.66	
	24	5d			5.8	2.2	0.7	
	25	5a, b, c		2.85($\lvert x_A\rvert < 0.94$)	5.9	2.24	0.71	
				(using (9))				
$\pi^+p \to p\pi^-\pi_s^+\pi_f^+$	8	4a	$M_{(\pi^+\pi^-)_{slow}} < 0.65$ GeV	(see figs. 9b and 12)	4.7	1.25	0.4	• s not high enough for $Z_A + Z_B$ much bigger than 4.6
and	11	4b			5.0	1.48	0.47	
$\pi^-p \to p\pi^+\pi_s^-\pi_f^-$	16	4a, b			5.4	1.80	0.57	
$\pi^-p \to p\pi^+\pi_s^-\pi_f^-$	25	4c	$M^2_{(3\pi)} \geqslant 2$ GeV^2 $M^2_{(p\pi^+\pi_s^-)} \geqslant 4$ GeV^2	$Z_A \leqslant 3.20(\lvert x_A\rvert < 0.96)$ $Z_B \leqslant 2.5(\lvert x_B\rvert < 0.92)$ (using (9))	5.9	2.24	0.71	• Few events of the DPE region are selected.
$\pi^-p \to p\pi^+\pi_s^-\pi_f^-$	205	4d	(1) $y_{AX} \geqslant 2$ $y_{BX} \geqslant 2$	$Z_A > 3.0(x_A > 0.950)$ $Z_B > 2.0(x_B > 0.86)$	7.9	6.22	1.97	• not symmetric cut as in the present analysis. Nevertheless, cross-sections of both selections are in agreement

Table 2 (*continued*)

Reaction	P_{lab} (GeV/c)	Ref.	Cuts used for (DPE) selection: From the authors	Cuts used for (DPE) selection: Equivalent in our Set of variables	Kinematic Boundary $\ln(s/s_0)$	Z_A and $Z_B \geqslant 2.3$: Maximum value of $M_{XA}(M_{XB})^*$	Z_A and $Z_B \geqslant 2.3$: M_X^{**}	Comments
			(2) A pion-pole dominance model [17] uses $M^2_{p\pi} \geqslant 4\ \mathrm{GeV}^2$ $M^2_{\pi^+\pi^-_{fast}} \geqslant 2\ \mathrm{GeV}^2$ (or $y_{\pi^-\pi^-} > 2$)	$Z_A \geqslant 3.08(\lvert x_A\rvert > 0.954)$ $Z_B \geqslant 2.64(\lvert x_B\rvert > 0.93)$ using (28), s independent				
$pp \to p_s\pi^+\pi^-p_f$		5e	(1) $M^2_{p\pi} \geqslant 4\ \mathrm{GeV}^2$, according to a pion pole dominance model [17] (2) identical to (1) above plus $M_X < 0.6$	$Z_A = Z_B \geqslant 3.08$ $(\lvert x_A\rvert > 0.954)$				• the events selected belong to the (DPE) region and cross-sections are in agreement with the prediction of a pion pole dominance model [17] and the present analysis • cut on M_X drastic as far below beginnning of (DPE) region

* = $\sqrt{s/10}$ according to (9) and (12).
** = $\sqrt{s/100}$ according to (35).

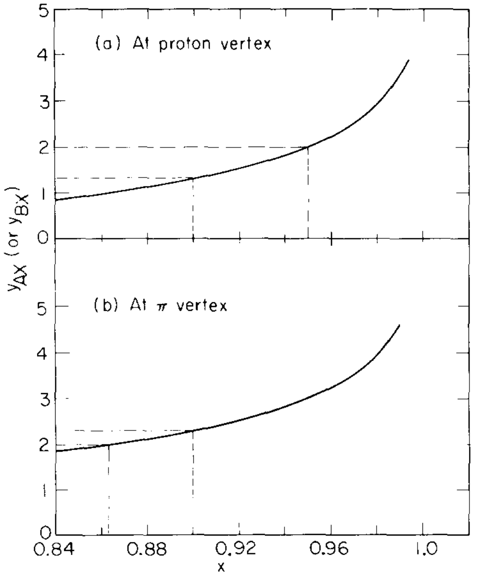

Fig. 13. y_{AX} (or y_{BX}) versus $|x|$. (a) at the proton vertex, (b) at the π vertex.

($0.9 \leqslant |x_{A,B}| \leqslant 1$, independent of the particle A or B considered) is that y_{AX} (and y_{BX}) have a different dependence on x at the π-vertex and the proton vertex, as illustrated in fig. 13. But though the criteria Z_A(and Z_B) $\geqslant 2.3$ do not select the same (DPE) candidates (which happen to be more in the π-diffraction region and less in the proton diffraction region), the cross sections corresponding to both selections (30 ± 11 μb in sect. 4, compared to 45 ± 13 μb evaluated from ref. [4d]) are compatible within the statistics.

(iii) Selections based on theoretical models. There are three experiments (of refs. [4c–d, 5e]) based on two different models (corresponding respectively to refs. [4c, 17]) which all use mass cuts either on $M_{AX}(M_{BX})$ or on $M_{A\pi}(M_{B\pi})$.

The selection of ref. [4c] on $M_{XA}(M_{XB})$ is equivalent to choosing $|x_A(x_B)| \leqslant 0.96$ (0.91) in a π–p experiment at 25 GeV/c. Most of the phase space so selected thus lies outside the double-pomeron region. The absence of events is thus meaningless.

A different criterion based on a pion-pole dominance model [17] uses a selection on $M_{A\pi}$ and $M_{B\pi}$. Though the corresponding constraints on x_A and x_B are different from ours ($0.95 \leqslant |x_p| \leqslant 1.$and $0.92 \leqslant |x_{\pi\,\text{fast}}| \leqslant 1.$), the cross sections for such selected events agree with the prediction of the model in the 205 GeV/c π^-p and pp experiments [4d, 5e].

In conclusion, only the two experiments [4d, 5e] performed at 205 GeV/c were at high enough energy to offer a chance for (DPE) events to be observed. Furthermore we have seen that (DPE) study not only requires high energies – typically NAL or ISR experiments – but also high statistics to permit the analysis of sect. 6.

8. Summary and conclusions

On the basis that the most satisfactory criterion for single-exclusive pomeron exchange (single diffraction) relates to a ratio of the missing mass to total energy, we have proposed a corresponding criterion for double-exclusive pomeron exchange in terms of two simultaneously measurable ratios. Multi-Regge models [12, 13] allow a triangle-plot analysis of the dependence in these ratios, and it has been shown that measurements over the range of energies available at NAL will allow decisive tests of the double-pomeron hypothesis. At the same time, we have demonstrated that measurements to date, when analyzed through the triangle plot, still have inadequate statistics within the region of relevance to double-pomeron exchange. The presence or absence of the double (exclusive) pomeron mechanism currently remains an undecided question.

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